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Full Length Research Paper

A Mathematical Model of Endemic Corruption, the Nigerian Perspective

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In this work, we created and examined a mathematical model that uses a system of nonlinear ordinary differential equations to investigate the dynamical aspects of corruption as a disease. The Jacobian matrix approach is used to examine the stability analysis of the corruption model. We found and examined the corruption-free equilibrium point. The domain where the model is well posed both mathematically and epidemiologically was identified. The basic reproduction ratio was then calculated using the next generation matrix. When the reproduction number is smaller than one, the model's corruption-free equilibrium is locally asymptotically stable. Additionally, we used MatCAD 14 algebra software to conduct numerical simulations. The mathematical model generated a population that is asymptotically stable, meaning that corrupt practices gradually disappear from the population. Additionally, the sensitivity of the model's parameters was calculated; sensitivity indices with negative signs show that the value of R0 falls as the parameters increase, while sensitivity indices with positive signs show that the value of R0 rises as the parameters increase. According to the sensitivity analysis, the parameters with the highest sensitivity are μ, α, β, θ, v, δ, and τ, in that order.

Keywords: Corruption, Nigeria, Mathematical modelling, Endemic and Sensitivity Analysis

INTRODUCTION

The misuse of authority for personal benefit is the standard definition of corruption. Bribery, embezzlement, nepotism, favouritism, fraud, and other dishonest or unethical actions by those in positions of power, especially in government or public organisations, are just a few examples of the various ways that this abuse might manifest. Transparency International defines corruption as when public servants abuse their power for their own gain, undermining institutions, eroding confidence, and impeding justice and progress. Corruptions can range from:

Petty Corruption: Is the regular misuse of authority by low-ranking officials, such as requesting bribes for necessities.

Grand Corruption: High-ranking officials' actions that skew laws or the way the government operates are known as.

Systemic Corruption: When corruption permeates an organization's or nation's culture and becomes ingrained in its system. Examples:

- When a police officer takes a bribe to ignore a traffic infraction.
- A public official giving contracts to family-owned businesses.
- The theft of public funds intended for infrastructure by a politician.

One of the main problems to Nigeria's political stability, economic progress, and effective governance is still corruption. Corruption persists in many areas of public life despite efforts to combat it through anti-corruption organisations and structural changes.

Nigerian corruption can be emphasised under:

1. The Corruption Perception Index (CPI) by Transparency International, Nigeria ranked 145th out of 180 countries and received a score of 25 out of 100 in Transparency International's 2023 CPI, indicating a high perception of corruption in the public sector.

In comparison, Nigeria scored 24/100 in 2022, ranking 150th, which shows only a marginal improvement. A score below 30 typically signifies widespread institutional corruption and a lack of effective governance mechanisms.

Nigeria, in contrast, only slightly improved, ranking 150th in 2022 with a score of 24/100. A score of less than 30 usually indicates a lack of efficient governance processes and pervasive institutional corruption.

2. Economic Cost of Corruption

According to estimates from the African Union (AU), corrupt practices and illegal financial flows cost Nigeria more than $18 billion a year.

• This drain impacts vital industries where money intended for development is redirected, such as infrastructure, health, and education.

• The Nigerian Economic Summit Group claims that corruption raises the cost of government contracts by 20% to 25%.

3. Public Perception and Experience

According to a 2019 poll conducted by the Nigerian Bureau of Statistics and the United Nations Office on Drugs and Crime (UNODC 2017), one in three Nigerians has paid a bribe over the preceding 12 months. The two industries where bribery was most commonly recorded were the police and public utilities.

• For routine services that ought to be provided at no cost, the average bribe paid was approximately ₦5,754 (roughly $16 at the time).

Nigerian corruption has proven to be a complicated and enduring problem that defies conventional anti-corruption tactics like awareness campaigns, institutional restructuring, and legal reforms. Even though these initiatives are still very important, they frequently lack the depth of analysis needed to comprehend the complex dynamics and dissemination of corrupt practices throughout different spheres of society.

In order to bridge this gap, mathematical modelling shows up as a potent and novel instrument for assessing, forecasting, and eventually managing corruption. Mathematical models enable researchers and policymakers to use precise quantitative methods to study the causes, transmission pathways, and potential control mechanisms of corruption by considering it as a dynamic system, akin to the spread of a contagious disease. In mathematical modelling, real-world phenomena are represented in a simplified but analytically tractable manner using equations, variables, and parameters. These models assist in identifying important variables, such as the rate of corruption, the level of enforcement, and reform initiatives.

Model diverse situations and forecast results based on different approaches.

Using mathematical and computer models, a number of scholars have tried to comprehend and manage corruption within the last ten years. Even though these studies have yielded insightful information, the majority of earlier models are still theoretical or narrowly focused, frequently lacking in local contextualisation, realism, or compartmental depth. This section highlights the study's uniqueness and significance by contrasting it with noteworthy earlier efforts.

To investigate corruption suppression, Athithan *et al*. (2018) created a SIR-type corruption model with optimal control. Without taking into account actual socio-political or economic structures unique to any one nation, the model viewed corruption as a purely biological epidemic.

- Hathroubi and Trabelsi (2014) described the spread of corruption and its effects on the economy using a bio-economic analogy. Despite its insightfulness, the study ignored reformation and punitive classes within the population and relied on abstract economic assumptions. Yusuf (2016) used differential equations to propose a social media-based approach for reducing corruption in Nigeria. Did not incorporate stability analysis or sensitivity indexing, and it was solely focused on media influence, with no compartmental detail such as immunity or reformation.
- This study, which uses information from Transparency International, EFCC reports, and local behavioural trends, is customised to the socio-political context of Nigeria, in contrast to generic models. It takes into account particular elements like judicial inefficiencies, public awareness, and political meddling.

This study presents a model with five components: The SICPR Model

- **S**usceptible
- **I**mmune
- **C**orrupt
- **P**unished
- **R**eformed

This structure reflects real-life corruption progression in governance and society—something absent in most existing models.

Most previous models ignore rehabilitation or punishment feedback into the corruption cycle. This study includes:

- Punished individuals who may reform,
- Reformed individuals who may relapse, and
- Rates of public enlightenment, modeling societal and institutional efforts.

Rigorous Mathematical Techniques

- **Equilibrium analysis** (corruption-free and endemic states),
- **Stability via Jacobian and eigenvalues**,
- **Basic reproduction number (R_0)** using the Next Generation Matrix, and
- **Sensitivity analysis** to determine which parameters most influence corruption levels.

Visualization and Simulation

Unlike previous models that frequently end at theoretical derivation, the model is numerically simulated using MathCAD, displaying the time-evolution of each compartment and illustrating how corruption can decay asymptotically under specific conditions. Although previous models established a significant foundation, they were mainly devoid of behavioral feedback, contextual realism, and simulations focused on policy. What sets this study apart is: capturing the entire corruption lifecycle (from vulnerability to reformation), simulating corruption as a contagious social phenomenon, and putting up workable solutions grounded in mathematical analysis. Therefore, this approach's uniqueness resides in its practical structure, local relevance, and ability to combine mathematical rigor with real-world policy ramifications.

Okwuagbala (2018) believes that bribery and other forms of petty or bureaucratic corruption, which are common and detrimental, have given way to a new form of corruption in the public sector. This erodes the moral values of the populace as well as the democratic and financial aspects of the government. Similar to corruption of any kind.
The effectiveness and integrity of public service delivery depend heavily on the recruitment and selection procedures. However, systemic corruption and socio-political biases frequently erode these processes in Nigeria. Omisore and Okofu (2014) claim that the Nigerian public service's hiring procedure is seriously defective and that political favouritism, religion, and ethnicity are often sacrificed for merit-based selection.

• The authors argue that the implementation of the Federal Character Principle, although originally intended to ensure equitable representation across regions and ethnic groups, has instead facilitated the entrenchment of nepotism and compromised professionalism.
Their study's empirical results show that corrupt practices and unofficial networks account for a sizable percentage of public service recruitment. In particular, 28% of those who were hired were said to have received help from friends or family, and others were hired through direct bribery or political connections. This trend highlights the supremacy of patronage systems over objective criteria like qualification, competence, or experience and exposes a chronic disrespect for open hiring practices.

Furthermore, nearly half of all public sector hires over the past three years were influenced by either nepotism, bribery, or both, highlighting the pervasive nature of corruption in Nigeria's civil service employment mechanisms. This has far-reaching implications, not only for the quality of public administration but also for national development, as unqualified or underqualified individuals occupy roles crucial to public service delivery.

The study by Omisore and Okofu serves as a stark reminder that without comprehensive reforms in the recruitment process, including strict enforcement of merit-based selection and insulation from political interference, Nigeria's public sector will continue to be plagued by inefficiency, low morale, and public distrust.

According to Ethelbert (2016) in the qualitative approach to the challenges of managing corruption in Nigeria by economic financial crimes commission, the study shows that the Commission is facing a lot of obstacles interfering with Its activities by the power that be these includes the unwholesome judicial process, elites, job insecurity ,weak anti-graft laws; The study recommended the creation of a court for the commission to handled their affairs; giving the commission the much needed autonomy it required, that will be totally independence from all form of Legislative and Executive influence among others.

Shiyanbade et al (2017) Using content analysis, study the efficiency and impact of anti-corruption institutions in corruption control in Nigeria. The study show that institutional weaknesses and system failure are responsible for corruption in Nigeria due to government interference in influencing the activities of this anti-corruption agency. The findings also revealed that, even though efforts were geared towards the prosecution of corrupt individuals and agencies, enduring policies for the prevention of these negative activities still remained. The study recommends that the anti-corruption agencies be granted autonomy and also total overhauling of the agencies with renew policies to foster the success of the fight against corruption in Nigeria.

According to Ocheci and Nwankwo (2012) study how effective the fight of EFCC and ICPC against corruption is promoting good governance in Nigeria find out that much is yet to be done and in order to redeem their image and gain trust form the Nigeria society at large, there is an urgent need for both EFCC and ICPC to live up to expectation summon the interest arrest and deal with culprit in respective on their places in the society. The study also recommends that the agencies be properly funded and the modus operandi of EFCC should change.

METHODOLOGY

Stability

When E. Torricelli investigated the equilibrium of a rigid body under the force of gravity in 1644, the idea of stability essentially arose from the study of an equilibrium state of a mechanical system. There have been numerous successful advancements and developments over the lengthy history of the evolution of the basic principles of systems and trajectory stabilities (Chen, 2014).

In system engineering, stability is crucial, particularly when it comes to automation and control systems. It is essential to its practical utility in terms of dynamics, control, and disturbance inputs, especially in the majority of real-world applications. Furthermore, stability necessitates that the internal signals and system outputs are contained within allowable bounds (also known as bounded-input/bounded output stability). Typically, stability issues are associated with differential equations of the following type

$$\dot{x} = f(t,x), \ \ x(t_0) = x_0$$

We may assume that the origin $x=0$ is an isolated equilibrium (or the null solution or the trivial solution) and $f(t,0) = 0$. We wish then to inquire about the dynamical system whether a slight perturbation from equilibrium will produce motion that will return to equilibrium or remain in the small neighbourhood or diverge all together from equilibrium. This situation brings about the idea of;

1. Stable, i.e. motion remains in a fixed orbit within a finite range of distance away from the critical point.
2. Asymptotically stable, i.e. converges to the critical point (equilibrium).
3. Unstable, i.e. moves away from the critical point.

Stability Basic Definitions

We consider nonlinear time-invariant system $\dot{x} = f(x)$, $f: \mathbb{R}^n \to \mathbb{R}^n$ a point $x\epsilon = \mathbb{R}^+$ is an equilibrium point of the system if $f(x_\epsilon) = 0$. We remark that $x_\epsilon$is an equilibrium point if and only if $x(t) = x_\epsilon$is a trajectory.

1 Definition: An equilibrium solution $x = x_\epsilon$ of $\dot{x} = f(x)$ is said to be:

(i) *stable* if, given any $\epsilon > 0$ and any $t_0 \geq 0$, there exists a $\delta = \delta(\epsilon, t_0)$ such that

$$\|x(t_0) - x_\varepsilon\| < \delta \Rightarrow \|x(t_0) - x_\varepsilon\| < \epsilon, \forall t \geq t_0 \geq 0$$

(ii) *uniformly stable* if, for every $\epsilon > 0$, there exists $\delta = \delta(\epsilon)$, independent of, t_0, such that for all $t_0 \geq 0$,
(iii) *unstable* if it is not stable.
(iv) *asymptotically stable* if there exists a $\delta = 0$, such that

(v) $$\left\|x(t_0) - x_e\right\| < \delta \ \Rightarrow \ \lim_{t\to\infty} x(t) = x_e$$

(vi) The system is *globally asymptotically stable* (G.A.S.) if for every trajectory $x(t)$ and $x(t) \to x_\epsilon$ as $t \to \infty$ (implies is the *unique* equilibrium point).
(vii) The system is *locally asymptotically stable* (L.A.S.) near or at $x_\epsilon$if there is an $R > 0$ s.t. $\|x(t_0) - x_\epsilon\| \leq R \Longrightarrow x(t) \to x_\epsilon$ as $t \to \infty$

Summary of Stability Classification Using Eigenvalues

Asymptotically stable: This implies that all solutions converge to the critical point as $t \to \infty$. A critical point is asymptotically stable if all of *A*’s eigenvalues are negative, or have negative real parts for complex eigenvalues.

➢ **Unstable** – All trajectories (or all but a few, in the case of a saddle point) start out at the critical point at $t \to \infty$ then move away to infinitely distant out as $t \to \infty$. A critical point is unstable if at least one of *A*’s eigenvalues is positive, or has positive real part for complex eigenvalues.

➢ **Stable** (or **neutrally stable**) – Each trajectory moves about the critical point within a finite range of distance. It never moves out to infinitely distant, nor (unlike in the case of saddle point).

Frequently the terms spiral sink and spiral source are used to refer to spiral points whose trajectories approach, or depart from, the critical point.

The Basic Reproduction Ratio, R_o

The expected number of secondary cases that would result from one typical infection joining in a population that is fully susceptible during its infectious period is known as the basic reproduction number, or R0 (Heffernan, Smith and Wahl, 2005). The sickness would eventually disappear when Ro < 1. A pandemic could result from the sickness spreading throughout the population if Ro > 1.

The next generation matrix is a technique used in epidemiology to calculate the basic reproduction number, or Ro, for a compartmental model of infectious disease transmission. The dominant eigenvalue, or the eigenvalue with the largest real part, of the next generation matrix, G, at the disease-free equilibrium (DFE), can be solved methodically to determine the R_0, according to Van den Driessche and Watmough (2002), Diekmann et al., (2009), and Diekmann et al., (1990). The existence and stability of the DFE for our epidemic model can be demonstrated mathematically (2).

For the next generation matrix $\boldsymbol{G = FV^{-1}}$, the *F* is the new infection (or transmission) matrix and *V* is the infection transfer (or transition) matrix. The entry of *i*th row and *j*th column of matrix *F* is denoted by F_{ij}, and $F_{ij} = {\partial \mathfrak{I}_i}/{\delta x_j}$ where $\mathfrak{I}_i$ is the *i*th equation of $\mathfrak{I}$ and x_j is the *j*th variable of the vector of infected classes. Similarly, the entry of *i*th row and *j*th column of matrix *V* is denoted by V_{ij} , and $V_{ij} = {\partial R_i}/{\delta x_j}$where R_i is the *i*th equation of R_0 and x_j is the *j*th variable of the vector of infected classes. The vector ʒ is the transmission rates' vector quantity, i.e., the changing rates from infected to non-infected classes, and vector *V* is the transition rates' vector quantity, i.e., the changing rates from infected to non-infected classes, and vector *V* is the transition rates' vector quantity, i.e., the changing rates among infected classes. The *F* is the Jacobian matrix of *F*, and *V* is the Jacobian matrix of *V*.

The basic reproduction ratio (or number) denoted by $\boldsymbol{R_o}$ is the average number of secondary infections caused by an infectious individual during his or her entire period of infectiousness (Diekmann *et al., 1990*). The basic reproduction ratio is an important non-dimensional quantity in epidemiology because it sets the threshold in the study of any given disease, both for predicting its outbreak and for evaluating its control strategies.

The Next Generation Matrix Approach

In general, $\boldsymbol{R_o}$ is found through the study and computation of the eigenvalues of the Jacobian matrix at the disease- or infectious-free equilibrium. Diekmann *et al., 2009* follow a different approach which is *the next generation matrix* method. This procedure converts a system of ordinary differential equations of a model of infectious disease dynamics to an operator (or matrix) that translate from one generation of infectious individuals to the next. The basic reproductive number is then defined as the spectral radius (dominant eigenvalue) of this operator. The basic reproduction ratio is obtained by taking the dominant eigenvalue of

The next-generation matrix FV^{-1} is given by the product of the Jacobian matrix of the new infection terms F_i, evaluated at the disease-free equilibrium E_0, and the inverse of the Jacobian matrix of the transition terms V, also evaluated at E_0,

Where F_i is the rate of appearance of new infection in compartment *i*, V_i is the transfer of infections from one compartment *i* to another and E_0 is the disease-free equilibrium.

Where F_i is the rate of appearance of new infection in compartment *i*, V_i is the transfer of infections from one compartment *i* to another and E_0 is the disease-free equilibrium

Model formulation

In this section, we propose an SICPR type model to analyze the dynamics of the endemic corruptions. Here the total population is divided in to five compartments: S denotes Susceptible Class; I represent Immune Class; C represents Corrupt Class; **Punitive Class** (*P*): These are class individuals who have been convicted or punished of corrupt practices and removed from office for a specific period of time (suspension) during which he/she cannot be involved in any corrupt act and cannot influence others during the suspension period. And lastly R represents the **Reformed Class** This class consists of the ex-convicts who have been reformed while serving their punishment and can become susceptible to corruption. Thus, the total population at time t is

$$N(t) = S(t) + I(t)) + C(t) + P(t) + R(t)$$

In our model, the parameter According to Eguda et al., (2017), the immune class I is made up of people who have moral standards from their homes and can never become corrupt at a rate (1 – θ)β, while the susceptible class S is created by the daily recruitment of people who were born into homes with good moral standards and are susceptible to being infected by the corrupt practices at a rate θβ. Corrupt individuals are punished at a rate δ. Susceptible people become corrupt at a rate of α after catching corruption infections or tendencies from corrupt people, thus switching from the susceptible class to the corrupt class.

The reformed individual leaves the reformed class into the susceptible class, while the susceptible-immune individual leaves the susceptible class for the immune class. The immune class is abandoned by the corrupted immune individual in favour of the corrupt class. Following public enlightenment, a corrupt individual transitions from the corrupt class to the reformed class. When a corrupt individual is convicted and imprisoned, they leave the corrupt class for the punished class P. They undergo reform while still serving their punishment at the rate ρ. While undergoing their penalty at rates τ and ρ, respectively, corrupt and punished people undergo reform in the reformed class R.

While vulnerable people who are prone to corruption become immune at a rate υ as a result of moral and religious convictions as well as public education campaigns, members of the reformed class eventually become susceptible at a rate ω. All the classes are subjected to natural exit from the endemic corruption system at a rate μ. Although μ is interpreted as natural death in (Binuyo 2019), we consider a natural exit due to retirement or illness in our analysis because no one can hold office indefinitely. The above model descriptions are illustrated in Figure 1.

The resulting mathematical model is given by a system of nonlinear ordinary differential equations;

$$\begin{cases} \dfrac{dS}{dt} = \theta\beta - \dfrac{\alpha S(t)C(t)}{N(t)} - (\mu + \nu)S(t) + \omega R(t), \\ \dfrac{dI}{dt} = (1-\theta)\beta + \nu S(t) - (\mu + \gamma)I(t) \\ \dfrac{dC}{dt} = \dfrac{\alpha S(t)C(t)}{N(t)} + \gamma I(t) - (\mu + \tau + \delta)C(t), \\ \dfrac{dP}{dt} = (t) - (\mu - \rho)P(t), \\ \dfrac{dR}{dt} = \tau C(t) + \rho P(t) - (\mu + \omega)R(t), \end{cases} \quad \text{…………(1)}$$

The model involves human population and hence all the parameters used are positive.

The descriptions of the model parameters are presented in Table 1.

Model Analysis

Further, we assume that the initial conditions

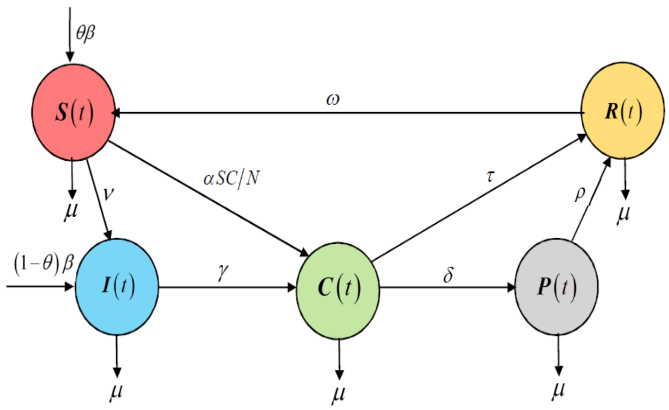

Figure 1. A schematic diagram of Nigeria corruption model

$S(0) \geq 0, I(0) \geq 0, C(0) \geq 0, P(0) \geq 0, R(0) \geq 0$
of the governing equations are non-negative.
In this section, we study the solution of the epidemiologically feasible region.

$$\Omega = \{S, I, C, P, R) \in R_+^5 : 0 \leq S(t) + I(t) + C(t) + P(t) + R(t) \leq N\}$$

Proposition: Assume that the initial conditions hold in R_+^5. Then the solutions

$\{S(t), I(t), C(t), P(t), R(t)\}$ of the model equations are non-negative for all $t \geq 0$.
Proof. Assume that all the state variables are continuous. Then, from the system of equations we can easily obtain:

$$\begin{cases} \frac{d}{dt}S(t) \geq -(\mu+\nu)S(t) \Longrightarrow S(t) = S(0)\ell^{-(\mu+\nu)t} \geq 0 \\ \frac{d}{dt}I(t) \geq -(\mu+\gamma)I(t) \Longrightarrow I(t) = I(0)\iota^{-(\mu+\gamma)t} \geq 0 \\ \frac{d}{dt}C(t) \geq -(\mu+\tau+\delta)C(t) \Longrightarrow C(t) = C(0)\ell^{-(\mu+\tau+\delta)t} \geq 0 \\ \frac{d}{dt}P(t) \geq -(\mu-\rho)P(t) \Longrightarrow P(t) = P(0)\ell^{-(\mu-\rho)t} \geq 0 \\ \frac{d}{dt}R(t) \geq -(\mu+\omega)R(t) \Longrightarrow R(t) = R(0)\ell^{-(\mu+\omega)t} \geq 0 \end{cases} \quad \ldots\ldots(2)$$

Hence, we can conclude that all the solutions are non-negative in for R^5 all $t \geq 0$.

1. Remark

The next proposition shows that it is sufficient to study the dynamics of our corruption model in a region Ω.

1. Proposition Assume that all the initial conditions are non-negative in R_+^5 for the system $\Omega = \{S, I, C, P, R) \in R_+^5 : 0 \leq S(t) + I(t) + C(t) + P(t) \leq N_0\}$
$N_0 = N(0)$, then the region Ω is positively invariant.

Table 1. Model parameters and their definitions

Parameter	Description
θ	Proportion of individuals not borne immune
β	Birth rate of individuals borne into the population
α	Effective corruption contact rate
γ	Rate at which susceptible individuals become immune to corruption
ω	Rate at which reformed individuals become susceptible to corruption
ν	Rate at which immune individuals are susceptible to corruption
δ	Rate at which prosecution and punishment of corrupt individuals occur
τ	Rate at which corrupt individuals become reformed due to public enlightenment
ρ	Rate at which punished individuals become reformed
μ	Natural exit from endemic corruption
$S(t)$	Susceptible individual class at time *t*.
$I(t)$	Immune individual class at time *t*.
$C(t)$	Corrupt individual class at time *t*
$P(t)$	Punished individual class at time *t*
$R(t)$	Reformed individual class at time *t*
$N(t)$	Number of Individuals in the population at time *t*
$s(t)$	Fraction of Susceptible Individual at time *t*
$i(t)$	Fraction of the Immune Individual at time *t*
$c(t)$	Fraction of the Corrupt Individual at time *t*
$p(t)$	Fraction of the Punished Individual at time *t*
$r(t)$	Fraction of the Reformed Individual at time *t*

Proof. We observe that all the state variables $S, I, C, P, R \in C(R^+, R^+)$ and the total population is

$$N(t) = S(t) + I(t) + C(t) + P(t) + R(t)$$

We have

$$= -\mu(S + I + C + P + R)$$

Hence

$$\frac{dN}{dt} = -\mu N \quad \Rightarrow \quad N(t) = N_0 e^{-\mu t}$$

It then follows that $N(t)$ is bounded for all $t \geq 0$. This implies that all the solutions of the model equation with initial condition in Ω remains in Ω. This completes the proof.

Corruption free equilibrium

We calculate the corruption – free equilibrium of model (1) by equating the right hand – side equations to zero and then putting

$$\frac{ds(t)}{dt} = \frac{di(t)}{dt} = \frac{dc(t)}{dt} = \frac{dp(t)}{dt} = \frac{dr(t)}{dt} = 0$$

At corruption free equilibrium, we must have $c^*(t) = p^*(t) = r^*(t) = 0$, hence we have

We obtain the following;

$$H_0 = (s^*, i^*, c^*, p^*, r^*) = \left(\frac{\theta\beta}{\mu+\nu}, \frac{\beta(\mu+\nu-\theta\mu)}{(\mu+\nu)(\mu+\gamma)}, 0,0,0\right)$$

The corruption compartment of model (1) consist of (C, P, R) classes. The basic reproduction number usually denoted R_0 is a threshold parameter defined as the average number of secondary infective produced when a single infected individual is introduced into a population consisting entirely of susceptible (Heesterbeek 2002). This is giving as;
$R_0 = \rho(FV^{-1})$ (the spectral radius of the Eigenvalues of the Jacobian matrix evaluated at the corruption free equilibrium point H_0). The Jacobian matrix calculated H_0 of the transmission terms F and that of the transition terms V respectively.

Thus;

$$F = \begin{pmatrix} \alpha s^* & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and}$$

$$V = \begin{pmatrix} (\mu+\tau+\delta) & 0 \\ -\delta & (\mu+\delta) \end{pmatrix}$$

Hence

$$FV^{-1} = \begin{pmatrix} \alpha s^* & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{1}{(\mu+\tau+\delta)} & 0 \\ \dfrac{\delta}{(\mu+\tau+\delta)} & \dfrac{1}{(\mu+\delta)} \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha s^*}{(\mu+\tau+\delta)} & 0 \\ 0 & 0 \end{pmatrix}$$

The basic reproduction ratio is given by the spectral radius of FV^{-1} i.e.

$$R_0 = \rho(FV^{-1}) = \frac{\alpha s^*}{(\mu+\tau+\delta)} = \frac{\alpha\left(\dfrac{\theta\beta}{\mu+\nu}\right)}{(\mu+\tau+\delta)}$$

By solving the dominant eigenvalues of the next generational matrix FV^{-1}, we get the basic reproduction number

$$\Rightarrow \quad R_0 = \frac{\alpha\theta\beta}{(\mu+\tau+\delta)(\mu+\nu)}$$

Remark 3

The basic reproduction number, denoted R_0 is threshold parameters defined as the average number of secondary infective produced when a single infected individual is introduced in to a population consisting entirely of susceptible individual. (Heesterbeek 2002).
Mathematically, R_0 is a threshold parameter for the stability of a diseases -free equilibrium and that can be used as an indicator for disease (corruption) control.

Local and global stability of CFE

In many epidemiology models there is a disease- free equilibrium point (DFE) at which the population remains in the absence of corruption. The following theorems discuss the local stability and global stability of DFE H_0.
The corruption -free equilibrium H_0 of model (1) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$ Where R_0 denotes the basic reproductive number,
F represents the matrix of partial derivatives of the rates secondary infections are produced and V represents the matrix of the expected time an individual initially introduced into the disease compartment (Van den Driessche and Watmough 2002). If $R_0 > 1$, the infected individual is infecting more than one further person, so the number of infective will exponentially increase and an epidemic will occur. However, if $R_0 < 1$, the infective is not passing the infection on to enough people to replace itself so the incident dies out and seize to persist in the community. There may be some secondary cases, but these will decrease with time and eventually the infection will become extinct. If $R_0 \approx 1$, the infection just barely succeeds in reproducing itself and there will be a similar number of cases at any later time (Binuyo 2015)

Hence the Jacobian matrix is given by

$$J_{sicpr}(H_0) = \begin{pmatrix} -(\mu+\nu) & 0 & \dfrac{-\alpha\theta\beta}{\mu+\nu} & 0 & \omega \\ \nu & -(\mu+\nu) & 0 & 0 & 0 \\ 0 & \gamma & \dfrac{-\alpha\theta\beta}{\mu+\nu} - (\mu+\tau+\delta) & o & 0 \\ 0 & 0 & \delta & -(\mu+\nu) & 0 \\ 0 & 0 & \tau & \rho & -(\mu+\nu) \end{pmatrix}$$

$$s^* = \frac{\theta\beta}{\mu+\nu}$$

$$i^* = \frac{(1-\theta)\beta+\nu s^*}{\mu+\gamma} = \frac{(1-\theta)\beta+\nu\left(\frac{\theta\beta}{\mu+\nu}\right)}{\mu+\gamma}$$

$$\Rightarrow \qquad i^* = \frac{\beta(\mu+\nu-\theta\mu)}{(\mu+\nu)(\mu+\gamma)}$$

Computation of the Characteristic Equation

The eigenvalues of the matrix $J_{s,i,c,p,r}(H_0)$ is given by the characteristic equation

$$\left|J_{s,i,c,p,r}(H_0) - \lambda I\right| = 0$$

$$\begin{bmatrix} -(\mu+v) & 0 & -\frac{\alpha\theta\beta}{\mu+V} & 0 & \omega \\ v & -(\mu+\gamma) & 0 & 0 & 0 \\ 0 & \gamma & \alpha-\frac{\alpha\theta\beta}{\mu+V}-(\mu+\tau+\delta) & 0 & 0 \\ 0 & 0 & \delta & -(\mu-\rho) & 0 \\ 0 & 0 & \tau & \rho & -(\mu+\omega) \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & \lambda \end{bmatrix} = 0$$

$$J_{sicpr}(H_0) = \begin{vmatrix} -(\mu+v)-\lambda & 0 & \frac{-\alpha\theta\beta}{\mu+v} & 0 & \omega \\ v & -(\mu+v)-\lambda & 0 & 0 & 0 \\ 0 & \gamma & \frac{-\alpha\theta\beta}{\mu+v}-(\mu+\tau+\delta)-\lambda & o & 0 \\ 0 & 0 & \delta & -(\mu+v)-\lambda & 0 \\ 0 & 0 & \tau & \rho & -(\mu+v)-\lambda \end{vmatrix}$$

$$\Rightarrow A\lambda^{5:} + B\lambda^4 + C\lambda^3 + D\lambda^2 + E\lambda + F = 0$$

The values of the calculated and estimated parameters are summarized in table 3. Ad its noted that the values of the parameters used in this work are obtained from estimation, model fitting and from the literature and unit of parameters (rate constants) is per mount. Using the parameter values in Table 2. The basic reproductive value $R_0 = 0.1432$ which is less than one

Mathcad Simulation of the Model Equations

Define a function that determines a vector of derivative values at any
solution point (t,Y):

$$D(t,Y) := \begin{bmatrix} \theta\cdot\beta - \alpha\cdot Y_0\cdot Y_2 - (\mu+\nu)\cdot Y_0 + \omega\cdot Y_4 \\ (1-\theta)\cdot\beta + \nu\cdot Y_0 - (\mu+\gamma)\cdot Y_1 \\ \alpha\cdot Y_0\cdot Y_2 + \gamma\cdot Y_1 - (\mu+\tau+\delta)\cdot Y_2 \\ \delta\cdot Y_2 - (\mu-\rho)\cdot Y_3 \\ \tau\cdot Y_2 + \rho\cdot Y_3 - (\mu+\omega)\cdot Y_4 \end{bmatrix}$$

Define additional arguments for the ODE solver:

Initial value of independent variable

$t0 := 0$ vector of initial function values

$t1 := 20$ Final value of independent variable

$$Y0 := \begin{pmatrix} 5 \\ 2 \\ 10 \\ 2 \\ 4 \end{pmatrix}$$

$num := 1 \times 10^3$
Number of solution values on [t0, t1

$S1 := \mathrm{Rkadapt}(Y0, t0, t1, num, D)$ Solution matrix	
$t := S1^{\langle 0\rangle}$	Independent variable values
$s := S1^{\langle 1\rangle}$	First solution function values
$i := S1^{\langle 2\rangle}$	Second solution function values
$c := S1^{\langle 3\rangle}$	Third solution function values
$r := S1^{\langle 4\rangle}$	Fifth solution function values

3. Remark

As we can see from the graphical profiles above, the mathematical model generated a population that is asymptotically stable, meaning that corrupt activities among the populous eventually disappear. As a result, the punished class shows an initial increase over time, but the population asymptotically decreases over an infinite period of time.

RESULTS AND DISCUSSION

Explanation of the Solution Matrix in table 3.
The solution matrix in table 3 is the output of the Ordinary Differential Equations Solver, which solves the system of differential equations representing the corruption dynamics It provides numerical solutions for the population in each compartment (S, I, C, P, R) over time.
These are calculated over the time interval [t0, t1] using initial values and parameter rates
Discussions from the above graphical profiles, we observe from Figure 2: Variation of the Susceptible Class was initially high but may fluctuate depending on:

Table 2. Parameter values

Parameter	Value	Reference
θ	0.45	Assumed
β	0.05	Assumed
α	0.234	Computed using TICPI (2017)
γ	0.25	Assumed
ω	0.14	Computed using TICPI (2017)
υ	0.212	Abdulrahman S (2014)
δ	0.26	Assumed
τ	0.15	Assumed
ρ	0.15	Assumed
μ	0.3	Assumed

Table 3. Solution Matrix

		0	1	2	3	4	5
	0	0	5	2	10	2	4
	1	0.03	4.61	2.006	10.4	2.069	4.002
	2	0.06	4.249	2.009	10.255	2.139	4.004
	3	0.09	3.915	2.01	10.345	2.21	4.007
	4	0.12	3.607	2.009	10.413	2.281	4.011
	5	0.15	3.323	2.006	10.46	2.352	4.016
	6	0.18	3.063	2.002	10.486	2.423	4.021
S1=	7	0.21	2.825	1.996	10.495	2.494	4.026
	8	0.24	2.607	1.988	10.486	2.564	4.031
	9	0.27	2.408	1.98	10.462	2.634	4.037
	10	0.3	2.226	1.97	10.424	2.704	4.042
	11	0.33	2.061	1.96	10.374	2.772	4.048
	12	0.36	1.91	1.948	10.312	2.84	4.054
	13	0.39	1.773	1.936	10.241	2.908	4.06
	14	0.42	1.648	1.923	10.16	2.974	4.065
	15	0.45	1.534	1.91	10.072	3.039	……..

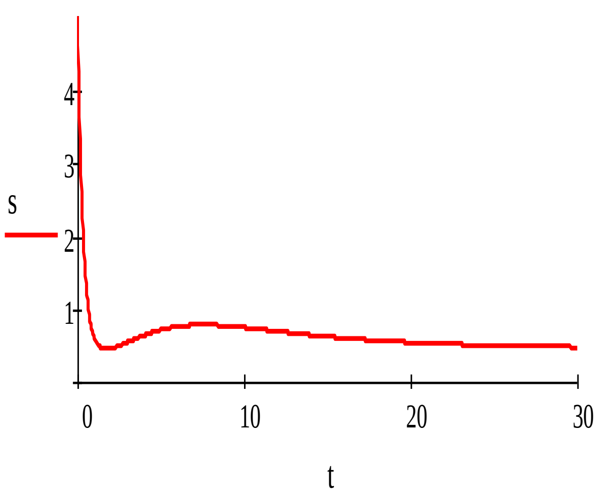

Figure 2. Variation of the susceptible class with time (months)

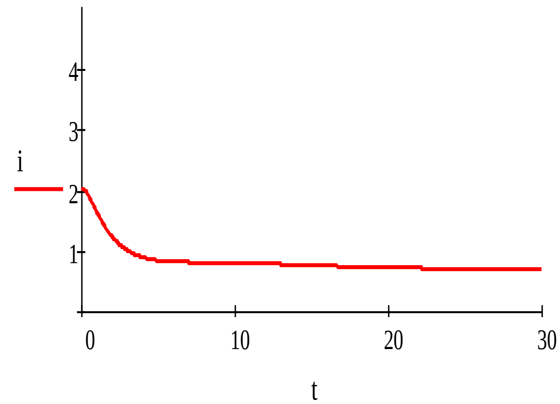

Figure 3. Variation of the immune class with time (months)

(i)New entries (birth/recruitment),
(ii)Transitions to corrupt or immune classes.
Interpretation

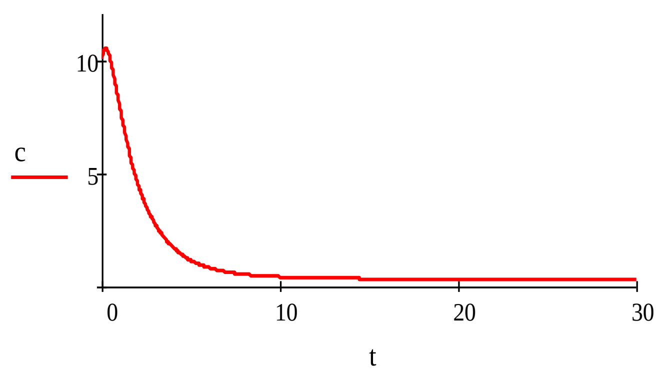

Figure 4. Variation of the corrupt class with time (months)

(i) The susceptible pool feeds corruption, so monitoring its size is essential.
(ii)Public policies should focus on reducing this pool via education and awareness campaigns.
Figure 3: Variation of the Immune Class

Observation:
(i)Shows a gradual increase in the number of immune individuals.
(ii)Over time, more susceptible individuals become immune due to public awareness, moral upbringing, or enlightenment programs.
Interpretation:
(i)Indicates success of preventive campaigns like ethics education, media awareness, and religious/moral influences.
(ii)A growing immune class contributes to long-term eradication of corruption.

Figure 4: Variation of the Corrupt Class
Observation:
(i) Initially high, but shows a decline over time.

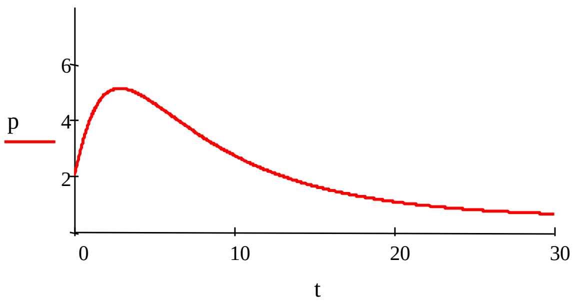

Figure 5. Variation of the punished class with time months)

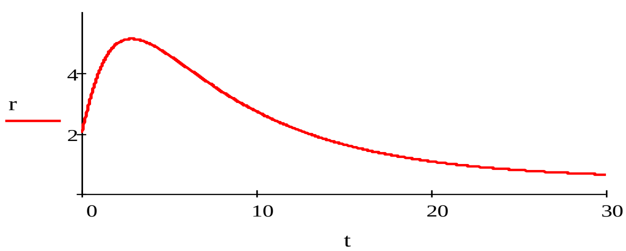

Figure 6. Variation of the reformed class with time (months)

(ii) The curve may rise slightly at the start, depending on initial conditions and contact rates, but eventually declines asymptotically.

Interpretation:

(i) Demonstrates that punitive and reformation strategies are working.

(ii) A sustained decrease implies that policy interventions (δ, τ, ρ, γ) are effective in reducing corruption.

(iii) Also shows how corruption does not vanish instantly, but requires time and consistent action.

Variation of the Punitive Class

Observation:

(i) Rises initially as more corrupt individuals get punished, then declines asymptotically as fewer people commit corruption or reform before punishment.

Interpretation:

(i) Shows short-term effectiveness of law enforcement.

(ii) Long-term decline may indicate either:

(iii) Fewer people becoming corrupt, or

(iv) Lax enforcement (if other variables show a rise in corruption).

Variation of the Reformed Class

Observation:

(i) Increases as punished and corrupt individual's reform.

(ii) May eventually stabilize or decline if reformed individuals return to the susceptible class (at rate ω).

Interpretation:

(i) Reflects the success or failure of rehabilitation programs.

(ii) High numbers mean many people can be turned away from corruption if properly enlightened and supported.

From figure 2 to figure 6 the mathematical model produced an asymptotically stable population such that corrupt practices among the populace die out from the populace as time progresses. Because of this the punished class depicts an initial rise over time, however this population declines asymptotically in infinite time.

Sensitivity Analysis

Understanding the relative contributions of the various model parameters that affect the occurrence and transmission of any disease is crucial. Sensitivity analysis is used in this study to quantify the effect of each input parameter on the value of an outcome and to identify key model parameters that will lessen the burden of the disease of corruption. The basic reproduction number, or R0, has a direct correlation with the initial corruption propagation. In order to determine the most important model, we conduct a sensitivity analysis on R_0 factors that are inherent to the dynamics of our corruption

To calculate the relative change in R0 to the relative change in the model-specific parameter c, we use the forward normalised sensitivity index of R_0. According to Chitnis *et al*., (2006) this is similarly defined using partial derivatives if R0 is a differentiable function of the model-specific parameter.

2 Definition

The normalized forward sensitivity index of a function $F(x_1, x_2, \dots x_n)$, $\forall x_i$ for ($1 \le i \le n$), is defined by

$$\Gamma_{x_i}^{F} = \frac{\partial F}{\partial x_i} \times \frac{x_i}{F}$$

We have

$$R_0 = \frac{\alpha\theta\beta}{(\mu+\tau+\delta)(\mu+\nu)}$$

To find sensitivity indices of R_0, we consider the intrinsic parameters; $\propto, \beta. \delta, \mu. \tau. and\ v$

Table 4. The normalized forward sensitivity indices of R_0 model parameters evaluated at the baseline values listed in Table 2.

4. Remark:

From Table 4. the sensitivity indices with negative signs indicate that the value of R_0 decreases when they are increasing, while the sensitivity indices with positive signs show that the value of R_0 increases when they are increasing. The sensitivity analysis shows that the most sensitive parameters are in the descending order of $, \alpha, \beta, \theta, v, \delta\ and\ \tau.$

A sensitivity index tells us how much a small change in one parameter (e.g., corruption rate, punishment rate) will affect the overall spread of corruption (measured by R_0). It answers:

"If this factor increases, will corruption get worse or better?"

Table 4. Summary of sensitivity indices and their values

Parameter	$\Gamma_{x_i}^{R_0} = \frac{\partial R_0}{\partial x_i} \times \frac{x_i}{R_0}$	Sensitivity Index Value
α	1	1
β	1	1
δ	$-\frac{\delta}{(\mu+\tau+\delta)}$	–0.366
μ	$-\frac{\mu(2\mu+\tau+\delta+\nu)}{(\mu+\tau+\delta)(\mu+\nu)}$	1.008
τ	$-\frac{\tau}{(\mu+\tau+\delta)}$	–0.211
θ	1	1
ν	$-\frac{\nu}{(\mu+\nu)}$	–0.414

Positive Sensitivity Index: Means if the parameter increases, R_0 (corruption spread) increases.
So corruption becomes harder to control.
Negative Sensitivity Index: Means if the parameter increases, R_0 decreases so corruption becomes easier to control from sensitivity results δ, τ corruptions will fade away with time rate at which prosecution and punishment of corrupt individuals and rate at which corrupt individuals become reformed decreases. i.e Negative sensitivity index

Summary

Nigeria, the most populous country in Africa, is highly ranked in corruption by Transparency International and other notable organizations that monitor corrupt practices around the world. High corruption cases had been linked to most Nigerians in foreign countries, so a lot of people have the perception that Nigerians are corrupt. In this research study, the stability analysis of the corruption mathematical model had been analyzed using the technique via the Jacobian matrix approach. The corruption free equilibrium point was obtained and analyzed. The basic reproductive number of the model was obtained to be the alternative way for identifying how the spread or the outbreak of the corrupt practices among the populace can be greatly reduced. It was observed that the mathematical model produced an asymptotically stable population such that corrupt practices among the populace die out from the populace as time increases when adequate measures mentioned in the recommendations are done by the right people and at the right time. The sensitivity of the parameters of the model was also computed and we were able to identify which of them were most sensitive to the corruption model dynamics.

CONCLUSION

Corruption undermines sound governance and prevents investment. Based on the governing mathematical model with a constant recruitment rate from the entire population, we created and examined a mathematical model in this research to investigate the dynamical nature of corruption. The domain where the model is well posed both mathematically and epidemiologically was identified. The basic reproduction number was then calculated using the next generation matrix. It was demonstrated that whenever the reproduction number is less than one, the corruption-free equilibrium of the model is locally asymptotically stable. Additionally, it is demonstrated that, as long as the fundamental reproduction number is bigger than one, the corruption epidemic equilibrium is locally asymptotically stable. Lastly, numerical simulations were used to confirm the analytical conclusion.

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